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A Veneziano model with cuts for $\overline{\mathrm{K}}$N and KN charge-exchange scattering

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Abstract. A Veneziano model, which agrees to leading order in s with the Reggeized U(6, 6) model, is applied to $\overline{\mathrm{K}}$N and KN charge-exchange scattering. Leading order duality-preserving' cuts are introduced and an improved agreement with low energy $\mathrm{K}^+\mathrm{n} \to \mathrm{K}^0\mathrm{p}$ data is obtained.

1. Introduction

Both the Reggeized U(6) × U(6) × O(3) (Hartley *et al* 1969, 1970, Collins *et al* 1970a, 1970b, 1970c) and the Reggeized U(6, 6) (Adjei *et al* 1970) absorption models have had considerable success in their application to both $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$ charge-exchange scattering and photoproduction. However, there was the usual lack of success in explaining the s dependence of the differential cross sections for $\mathrm{K}^+\mathrm{n} \to \mathrm{K}^0\mathrm{p}$ in the momentum range 2·3 to 5·5 GeV/c. The cause of this could have been the form of the high-energy Regge pole graph, the high-energy form of the cut or the interference effect between the two.

Recently, several authors (Inami 1969, Igi and Storrow 1969, Namyslowski *et al* 1970) have constructed models for $\overline{\mathrm{K}}$N and KN scattering, and, in particular Berger and Fox (1969) have applied a Veneziano pole model to both $\overline{\mathrm{K}}$N and KN charge-exchange forward scattering. They used the asymptotic form of the Veneziano model for $\overline{\mathrm{K}}$N charge-exchange but were unable to fit the low energy KN charge-exchange data without the introduction of many satellites.

However, Lovelace (1969) has proposed that cuts are more important than the first daughter and satellite terms (except of course for those with large residues). For example, in πN charge-exchange scattering, cuts are regarded as equivalent to a phenomenological ρ′ which is only half a unit of angular momentum below the ρ, while the first Veneziano f^0 daughter ρ′ is a complete unit down. This, together with the necessity to fill in nonsense dips make cuts necessary for agreement between theory and experiment. The introduction of cuts follows naturally from Lovelace's K matrix unitarization method (Lovelace 1969, Roberts 1970) equivalent to our method. Hence, we are led to construct a model suitable for both $\overline{\mathrm{K}}$N and KN charge-exchange forward scattering, which is normalized to agree to leading order in s with our Reggeized U(6, 6) model, has absorptive cuts, and uses the full Veneziano form in the $\overline{\mathrm{K}}$N charge-exchange amplitude.

In § 2, the Veneziano formalism for the pole graph is developed, followed in § 3 by a discussion on the introduction of imaginary parts in fermion effective trajectories and in § 4 the removal of lower lying parity doublets. § 5 gives the formalism for the absorptive cuts and we conclude with a discussion of results in § 6.

2. Veneziano formalism

We will write our formalism in terms of the invariant amplitudes A and B, defined by the $0^-\frac{1}{2} \to 0^-\frac{1}{2}^+$ M function

$$M = A + B\not{Q}$$

where Q is equal to one half the sum of the initial and final meson four momenta. Imposing exchange degeneracy on the t channel exchanges in our Reggeized U(6, 6) model as required by the absence of a nonexotic u channel in $\bar{\mathrm{K}}$N and a nonexotic s channel in KN, we may write

$\mathrm{K}^-\mathrm{p} \to \bar{\mathrm{K}}^0\mathrm{n}$

$$\begin{aligned} A(s,t,u) &= \frac{2gh}{m_1}\left(1+\frac{q_3^2}{2\mu_3^2}\right)\left\{\left(1+\frac{q_3^2}{2m_3\mu_3}\right)-\frac{5}{3}\left(1+\frac{2m_3}{\mu_3}\right)\right\}\Gamma(1-\alpha_t)\,\mathrm{e}^{-i\pi\alpha_t}(\alpha' s)^{\alpha_t} \\ B(s,t,u) &= \frac{20gh}{3}\left(1+\frac{q_3^2}{2\mu_3^2}\right)\left(1+\frac{2m_3}{\mu_3}\right)\left(1-\frac{M_1^2}{4m_1^2}\right)\alpha'\Gamma(1-\alpha_t)\,\mathrm{e}^{-i\pi\alpha_t}(\alpha' s)^{\alpha_t-1} \end{aligned} \tag{1}$$

$\mathrm{K}^+\mathrm{n} \to \mathrm{K}^0\mathrm{p}$

$$\begin{aligned} A(s,t,u) &= \frac{2gh}{m_1}\left(1+\frac{q_3^2}{2\mu_3^2}\right)\left\{\left(1+\frac{q_3^2}{2m_3\mu_3}\right)-\frac{5}{3}\left(1+\frac{2m_3}{\mu_3}\right)\right\}\Gamma(1-\alpha_t)(\alpha' s)^{\alpha_t} \\ B(s,t,u) &= \frac{20gh}{3}\left(1+\frac{q_3^2}{2\mu_3^2}\right)\left(1+\frac{2m_3}{\mu_3}\right)\left(1-\frac{M_1^2}{4m_1^2}\right)\alpha'\Gamma(1-\alpha_t)(\alpha' s)^{\alpha_t-1} \end{aligned} \tag{2}$$

where g is the universal U(6, 6) baryon–baryon–meson coupling constant determined by Chew–Low extrapolation

$$\frac{g^2\pi\mathrm{NN}}{4\pi} = 14{\cdot}9 = g^2\left(\frac{5}{3}\right)^2\left(1+\frac{2m_2}{\mu_2}\right)^2\left(1-\frac{\mu_1^2}{4m_1^2}\right)^2$$

and h is the numerical U(6, 6) meson–meson–meson coupling constant determined from the $\rho \to 2\pi$ decay width of the Novosibirsk colliding beam experiment giving

$$\frac{g^2\pi\pi\rho}{4\pi} = 2{\cdot}13 = \frac{h^2}{\pi}\left(1+\frac{q_2^2}{2\mu_2^2}\right)^2$$

and where q_i^2 are interpreted in terms of masses as the U(6, 6) currents were derived 'on-shell'. m_i, μ_i are baryon and meson masses respectively. We have assumed linear trajectories with a common slope as required by the Veneziano model.

Formulae (1) and (2) satisfy the requirements of duality as s channel resonances are only present in $\bar{\mathrm{K}}$N. This leads to the amplitude for $\bar{\mathrm{K}}$N being complex and for KN purely real.

A Veneziano formula which reproduces equation (1) to leading order in s in the asymptotic limit as $s \to \infty$, t fixed is:

$$A_{\bar{K}N}(s,t,u) = \Lambda_{A_1}\frac{\Gamma(1-\alpha_t)\Gamma(\frac{3}{2}-\alpha_s^{Y_0^*})}{\Gamma(\frac{3}{2}-\alpha_t-\alpha_s^{Y_0^*})}+\Lambda_{A_2}\frac{\Gamma(1-\alpha_t)\Gamma(\frac{1}{2}-\alpha_s^{Y_0^*})}{\Gamma(\frac{3}{2}-\alpha_t-\alpha_s^{Y_0^*})}$$
$$+\Sigma_{A_1}\frac{\Gamma(1-\alpha_t)\Gamma(\frac{3}{2}-\alpha_s^{Y_1^*})}{\Gamma(\frac{3}{2}-\alpha_t-\alpha_s^{Y_1^*})}$$

$$B_{\bar{K}N}(s,t,u) = \Lambda_{B_1}\frac{\Gamma(1-\alpha_t)\Gamma(\frac{1}{2}-\alpha_s^{Y_0^*})}{\Gamma(\frac{3}{2}-\alpha_t-\alpha_s^{Y_0^*}}+\Sigma_{B_1}\left(1-\frac{t}{N}\right)\frac{\Gamma(1-\alpha_t)\Gamma(\frac{3}{2}-\alpha_s^{Y_1^*})}{\Gamma(\frac{5}{2}-\alpha_t-\alpha_s^{Y_1^*})}$$

where numerically

$$\Lambda_{A_1} = \frac{2gh}{m_1}\left(1+\frac{q_3^2}{2\mu_3^2}\right)\left\{\left(1+\frac{q_3^2}{2m_3\mu_3}\right)-\frac{5}{3}\left(1+\frac{2m_3}{\mu_3}\right)\right\}\frac{\Lambda'}{\Sigma'+\Lambda'}$$

$$\Lambda_{A_2} = X\frac{(\Sigma'+\Lambda')}{\Lambda'}\Lambda_{A_1} \qquad \Sigma_{A_1} = \frac{\Sigma'}{\Lambda'}\Lambda_{A_1} \tag{3}$$

$$\Lambda_{B_1} = -\frac{20gh}{3}\left(1+\frac{q_3^2}{2\mu_3^2}\right)\left(1+\frac{2m_3}{\mu_3}\right)\left(1-\frac{M_1^2}{4m_1^2}\right)\alpha'\frac{\Lambda}{\Sigma+\Lambda}$$

$$\Sigma_{B_1} = \frac{\Sigma}{\Lambda}\Lambda_{B_1}$$

and where Σ', Λ', Σ, Λ, X and N are parameters, α_t is the exchange degenerate $\rho - A_2$ trajectory, and $\alpha_s^{Y_0^*}$ and $\alpha_s^{Y_1^*}$ are the exchange degenerate $\Lambda_\alpha - \Lambda_\gamma$ and $\Sigma_\beta - \Sigma_\delta$ trajectories respectively. The exchange degeneracy of the s channel fermion trajectories is implied by the absence of nonexotic u channel exchanges and in common with other authors (Inami 1969, Igi and Storrow 1969, Namyslowski *et al* 1970, Berger and Fox 1969, Lovelace 1969). We have neglected the $\Lambda_\beta = \Lambda_\delta$ and $\Sigma_\alpha = \Sigma_\gamma$ trajectories as they are weakly coupled to the $\bar{K}N$ channel. Two terms appear in equation (3) which do not give rise to terms in equation (1). The first is the term A which is proportional to

$$\Lambda_{A_2}\frac{\Gamma(\frac{1}{2}-\alpha_s^{Y_0^*})}{\Gamma(\frac{3}{2}-\alpha_t-\alpha_s^{Y_0^*})}$$

which is a satellite and is proportional to s^{α_t-1}. This term is included so that we have a $\Lambda(1115)$ pole in the amplitude, however in the energy range considered it is an order of magnitude less than the leading order Y_0^* term. The second is the term proportional to

$$\frac{t}{N}\frac{\Gamma(\frac{3}{2}-\alpha_s^{Y_1^*})}{\Gamma(\frac{5}{2}-\alpha_t-\alpha_s^{Y_1^*})}$$

in B. Its introduction is required so that the Y_1^* has a leading behaviour in s in the backward direction for $K^+n \to K^0p$. We choose N so this is suitably small in the forward scattering region.

The invariant amplitudes for $K^+n \rightarrow K^0p$ can be obtained from those of $K^-p \rightarrow \bar{K}^0n$ by s–u crossing. The u channel $K^-p \rightarrow \bar{K}^0n$ amplitude is just the time reversed s channel $K^+n \rightarrow K^0p$ amplitude. Hence we may write

$$\begin{aligned} A_{KN}(s, t, u) &= A_{\bar{K}N}(u, t, s) \\ B_{KN}(s, t, u) &= -B_{\bar{K}N}(u, t, s). \end{aligned} \tag{4}$$

3. Trajectories

As stated before, the Veneziano model requires that we use linear trajectories with a common slope. For fermion trajectories linear effective trajectories imply parity doubling by MacDowell symmetry. From the Chew–Frautschi plot, we may write for the exchange degenerate $\rho - A_2$ t channel trajectory

$$\alpha_t = 0{\cdot}44 + 0{\cdot}95t. \tag{5}$$

In order to give the resonances a finite width we have added an imaginary part to the trajectories above threshold. This imaginary part has been taken to be linear in s to avoid generating ancestors. Hence, the s channel fermion effective trajectories have the form:

$$\alpha_s = \alpha_0 + \alpha' s + i\alpha_I(s - s_0) \qquad \alpha_I \gtrsim 0 \tag{6}$$

while the u channel ones are

$$\alpha_u = \alpha_0 + \alpha' u$$

where α_I determines the size of the imaginary part and $s_0 = (m_p + m_K)^2$ is the s channel threshold. A simple model (Chew *et al* 1962) is used to find α_I. A Breit–Wigner form is assumed for the resonances and gives approximately the correct width. Near a pole above threshold, the partial wave amplitude can be written as

$$f_J \simeq \frac{r(s, J)}{J - \alpha_s}.$$

The resonance occurs at $J = \alpha_0 + \alpha' s_{\text{res}}$ on the real s axis. At half width

$$\begin{aligned} \alpha_I(s_{\text{res}} - s_0) &= J - \alpha_0 - \alpha' s_{1/2} = \alpha'(s_{\text{res}} - s_{1/2}) \\ \alpha_I &= \frac{\alpha' \Gamma m_{\text{res}}}{(m_{\text{res}}^2 - s_0)} \end{aligned} \tag{7}$$

α_I is averaged along each trajectory. Although strict crossing symmetry is destroyed, we only consider s channel resonance widths as this is the only channel above threshold.

In the asymptotic limit the imaginary part of the effective trajectory for the case of $\bar{K}N$ charge-exchange results in the leading terms in s being multiplied by $\{1 + i(\alpha_I/\alpha')\}^{\alpha_t}$. However, as α_I/α' is small this agrees to first order in a binomial expansion without Reggeized U(6, 6) formalism. However, if we did not have exchange-degenerate trajectories in our problem and so had explicit signature present, such factors would displace the nonsense dips and other conventional 'Regge features'. The parametrization of the fermion trajectories are given in tables 1 and 2.

Table 1. Parameters for Y_0^* trajectory

Resonance	J^P	Contribution to α_1
$\Lambda_\alpha(1115)$	$\frac{1}{2}^+$	below threshold
$\Lambda_\gamma(1520)$	$\frac{3}{2}^-$	0·075
$\Lambda_\alpha(1815)$	$\frac{5}{2}^+$	0·099
$\Lambda_\gamma(2100)$	$\frac{7}{2}^-$	0·084

$\alpha^{Y_0^*}(s) = -0{\cdot}67+0{\cdot}95s+\mathrm{i}\,0{\cdot}09\,(s-s_0)$

Table 2. Parameters for Y_1^* trajectory

Resonance	J^P	Contribution to α_1
$\Sigma_\beta(1385)$	$\frac{3}{2}^-$	below threshold
$\Sigma_\delta(1770)$	$\frac{5}{2}^-$	0·186
$\Sigma_\beta(2030)$	$\frac{7}{2}^+$	0·110

$\alpha^{Y_1^*}(s) = -0{\cdot}33+0{\cdot}95s+\mathrm{i}\,0{\cdot}148\,(s-s_0)$

4. Elimination of low lying parity doublets

Any models incorporating linear fermion trajectories are parity doubled. However, some of these parity doublets are not present in nature and their absence can be used to provide constraints on the parameters (Inami 1969, Igi and Storrow 1969, Namyslowski *et al* 1970, Berger and Fox 1969, Lovelace 1969). We will use the absence of the $\frac{1}{2}^-$ and $\frac{3}{2}^+$ particles on the Y_0^* trajectory to relate X and Σ'/Λ' respectively to Σ/Λ. This is done by using the convention that as the Y_0^* particles have parity $(-1)^{J-1/2}$ they are in the $J = l-\frac{1}{2}$ partial wave with mass $w = -m_{\text{res}}$, so that Y_0^* parity doublets lie in the same partial wave with $w = m_{\text{res}}$, where $w = \sqrt{s}$. The residue of the leading term in $\cos\theta$, where θ is the centre of mass (CM) scattering angle, in the partial wave amplitude

$$f_{l\pm} = \frac{1}{2}\int_{-1}^{+1} (f_1 P_l(\cos\theta)+f_2 P_{l\pm 1}(\cos\theta))\,\mathrm{d}(\cos\theta) \tag{8}$$

is then made to vanish at the mass of the parity doublet. The implication is that

$$f_1 = 0 \qquad \text{at} \qquad w = m_{\text{res}}$$

so

$$\frac{A}{B} = -(m_{\text{res}}-m_{\text{p}}). \tag{9}$$

The constraints on the parameters are given in table 3.

Table 3. Relations given by absence of parity partners

Parity partner	Relation obtained
$\Lambda(1115)\frac{1}{2}^-$	$X = -\dfrac{0{\cdot}362}{(\Sigma/\Lambda)+1}$
$\Lambda(1520)\frac{3}{2}^+$	$\dfrac{(\Sigma/\Lambda)+1}{(\Sigma'/\Lambda')+1} = 0{\cdot}829$

5. Absorptive corrections

To introduce these, we take the two independent s channel helicity amplitudes:

$$\langle 0\tfrac{1}{2}|\phi|0\tfrac{1}{2}\rangle = \frac{1}{4\pi w}\cos\frac{\theta}{2}\{m_1 A+(E_1 w-m_1^2)B\}$$
$$\langle 0\tfrac{1}{2}|\phi|0-\tfrac{1}{2}\rangle = \frac{1}{4\pi w}\sin\frac{\theta}{2}\{E_1 A+m_1(w-E_1)B\} \tag{10}$$

where E_1 and m_1 are the energy and mass of the target nucleon. These are then partial-wave analysed such that

$$\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle = \sum_J (2J+1)\langle\lambda_3\lambda_4|T^J(s)|\lambda_1\lambda_2\rangle d^J_{\lambda\mu}(\theta) \tag{11}$$

where $\lambda = \lambda_1-\lambda_2$, $\mu = \lambda_3-\lambda_4$ with $J = l+\tfrac{1}{2}$.

Using the orthonormality relation, we get

$$\langle\lambda_3\lambda_4|T^J(s)|\lambda_1\lambda_2\rangle = \tfrac{1}{2}\int_{-1}^{+1} \mathrm{d}(\cos\theta) d^J_{\lambda\mu}(\theta)\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle. \tag{12}$$

This is then modified according to the Watson prescription, giving the modified partial-wave amplitude

$$\langle\lambda_3\lambda_4|T'^J(s)|\lambda_1\lambda_2\rangle = \langle\lambda_3\lambda_4|S^{\mathrm{el}\,J}|\lambda_3\lambda_4\rangle\langle\lambda_3\lambda_4|T^J(s)|\lambda_1\lambda_2\rangle \tag{13}$$

where we have assumed that the elastic scattering matrix element $S^{\mathrm{el}\,J}$ is pure nonflip and that the final state elastic scattering is the same as that of the initial state. $S^{\mathrm{el}\,J}$ is parametrized by a real Gaussian, in the usual notation

$$S^{\mathrm{el}\,J} = 1-c\exp\left(\frac{-J(J+1)}{\nu^2 q^2}\right). \tag{14}$$

The modified partial waves are then resummed to give the modified helicity amplitudes $\langle\lambda_3\lambda_4|\phi'|\lambda_1\lambda_2\rangle$ which are then compared with experiment.

Although the introduction of a diffractive effect in both the initial and final states destroys crossing symmetry, the leading order cut is still 'duality preserving' (Krzywicki 1970). This is because the diffractive effect is equivalent to a fixed pole pomeron, which leaves an amplitude, such as $\langle K^0 p|\phi|K^+ n\rangle$, real after the introduction of the absorptive effect. For KN charge-exchange scattering, as no K^+n elastic scattering data exist, K^+p elastic scattering parameters are used on the assumption that they are not too different. The elastic scattering parameters are given in table 4.

Table 4. Absorption coefficients

Elastic scattering process	P_{lab}(GeV/c)	ν^{-1}(GeV)	c
	3·5	0·26	0·79
	5·0	0·26	0·74
K^-p	7·1	0·26	0·70
	9·5	0·26	0·67
	12·3	0·26	0·65
	2·3	0·38	1·00
K^+p	2·97	0·36	0·91
	5·5	0·31	0·68

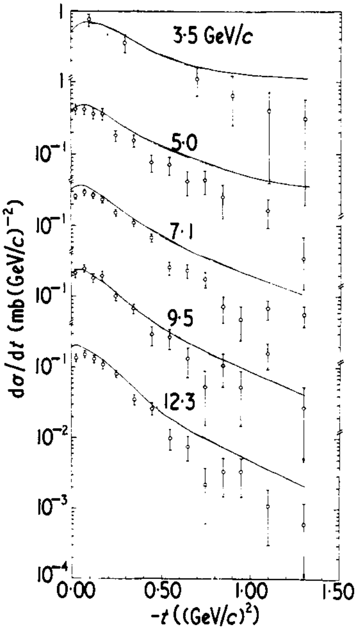

Figure 1. Differential cross sections for $K^-p \rightarrow \overline{K}^0n$. Data from Brody and Lyons (1966) and Astbury *et al* (1966).

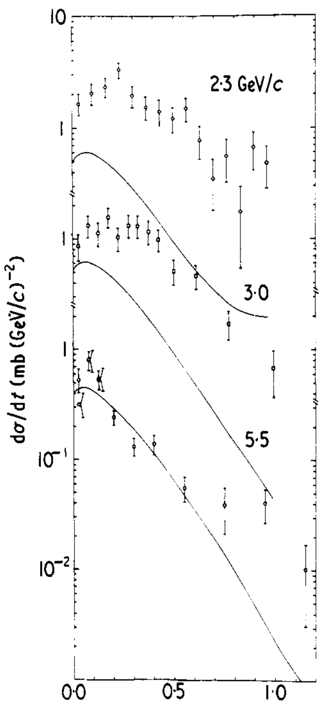

Figure 2. Differential cross sections for $K^+n \rightarrow K^0p$. Data from Butterworth *et al* (1965), Goldschmidt-Clermont *et al* (1968) and Cline *et al* (1969). The lower two energies do not include a deuteron correction. At 5·5 GeV/c, the forwardmost top three points include the deuteron correction.

6. Discussion and results

The mass splitting used to give the results was

$$m_1 = \langle \mathrm{SU(3)}\tfrac{1}{2}^+ \text{ octet} \rangle = 1{\cdot}15\ \mathrm{GeV}/c^2$$

$$M_1 = \langle \mathrm{SU(3)}1^- \text{ and } 2^+ \text{ nonets} \rangle = 1{\cdot}115\ \mathrm{GeV}/c^2$$

$$\mu_1 = \langle \mathrm{SU(3)}0^- \text{ nonet} \rangle = 0{\cdot}42\ \mathrm{GeV}/c^2$$

$$m_2 = m_3 = \langle \mathrm{SU(6)}\underline{56} \rangle = 1{\cdot}27\ \mathrm{GeV}/c^2$$

$$\mu_2 = \langle \mathrm{SU(6)}\underline{1}+\underline{35} \rangle = 0{\cdot}63\ \mathrm{GeV}/c^2$$

$$\mu_3 = \langle \mathrm{SU(3)}0^- \text{ nonet, } 1^- \text{ and } 2^+ \text{ nonets} \rangle = 0{\cdot}88\ \mathrm{GeV}/c^2.$$

The justification for this splitting is as follows. SU(6) masses were used in the coupling constants (except for the kinematic factor) as these are universal to the U(6, 6) currents and both 0^- and 1^- particles occur at the vertices. The factors $\{1-(M_1^2/4m_1^2)\}$ and $\{1-(\mu_1^2/4m_1^2)\}$ are essentially kinematic factors 'on-shell' and SU(3) masses provided the most effective group theoretic interpretation. M_1 was modified to include both the 1^- and 2^+ SU(3) nonets, to take account of the two exchange degenerate parents.

The fitting of the pole + cut amplitudes to the differential cross section data was done using MINUITS (CERN program library no D506) and $\Sigma/\Lambda = 0{\cdot}0423$. N was chosen to be 20 but the theoretical curve was remarkably insensitive to this parameter. The fit to the data gave a χ^2 of 617 for 101 data points. 30 partial waves and a 48-point gaussian quadrature proved to be adequate for convergence of the partial-wave expansion.

The $\bar{\mathrm{K}}\mathrm{N}$ exchange results (figure 1) were very good in the forward direction, but too large at wide angles, particularly at low energies, a feature in common with our previous absorptive Regge fits. This indicates that the cut has a significant contribution to this wide angle behaviour. As the cut is essentially parametrized at high energies, the continuation to lower energies may be somewhat questionable.

As far as the KN charge-exchange results (figure 2) are concerned, in addition to the comments above, we make the observation that although we have improved the s dependence slightly and the t dependence greatly, probably due to the inclusion of the u channel, we are still too low in normalization. This is a feature in common with the results of Berger and Fox which they overcame by the addition of many satellites.

Comparing the results for our parameters to those of Inami (1969) and Berger and Fox (1969) we find that we have a larger Y_0^* term, but a smaller Y_1^* term. However, this is not unreasonable as shown by Barger (1969). He successfully applied a conventional Regge fit to $\mathrm{K}^+\mathrm{p}$ backward elastic scattering just using a Y_0^* trajectory and neglecting the Y_1^* contribution because of SU(3) and dispersion relation arguments. The $\mathrm{K}^+\mathrm{p}$ elastic u channel is related to the $\bar{\mathrm{K}}\mathrm{N}$ charge-exchange s channel by isospin crossing, so giving an indication that the Y_1^* contribution could have a smaller value than previously used for forward scattering. $\mathrm{K}^+\mathrm{n}$ backward elastic scattering data would be useful for obtaining the Y_1^* contribution.

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